Chapter 7: Bending and Shear in Simple Beams

Introduction
A beam is a long, slender structural member that resists loads that are generally applied transverse (perpendicular) to its longitudinal axis.

- These transverse loads cause the beam to bend in the plane of the applied loads.
- Internal bending stresses and shear stresses are developed in the material as it resists these loads.
- Axial forces (and axial stresses) may also be present in the beam when the applied loads are not perpendicular to the axis.

Beams are a common type of structural member.
- Beams are used in the roofs and floors of buildings.
- Beams are used for bridges and other structural applications.

Beams are usually long, straight, prismatic (i.e. symmetrical) bars.
- Not all beams need to be horizontal; they may be vertical or on a slant.

Designing a beam consists essentially in selecting the cross section that will provide the most effective resistance to shear and bending produced by the applied loads.

7.1 Classification of Beams and Loads
Beams are often classified according to their support conditions. Following are six major beam types. The span “L” is the distance between supports.

Simply supported    Overhanging    Cantilever

Statically Determinate
**Type of Connections**

Actual support and connections for beams are idealized as rollers, hinges (pins), or fixed.

Following are examples of common support/connection conditions.

1. **Roller supports**

   This reaction is equivalent to a force with a known line of action (direction).

   - There is only one unknown involved with this support (i.e. the magnitude of the reaction).
   - This reaction prevents translation of the free body in one direction.
   - This reaction does not prevent the body from rotating about the connection.

**Examples of this type of support include the following.**

- A beam supported by a neoprene pad.
- A beam supported by a concrete or steel cylinder (i.e. a rocker).
2. Pin support

This reaction is equivalent to a force with an unknown line of action.

- There are two unknowns involved with this support.
- The magnitude and direction of the reactive force is unknown.
- Normally, we will work with the components of the unknown reactive force, thus fixing the directions; the magnitudes of the two components become the unknowns.
- This reaction prevents translation of the free body in all directions.
- This reaction does not prevent the body from rotating about the connection.

Examples of this type of support include the following.
- A timber beam/timber column connection with T-plate.
- A steel beam connected to a steel girder with a clip angle.
- A pin-connected column base.
- A gusset plate on a truss.
- A ball and socket joint.

3. Fixed support

This reaction is equivalent to a force (with an unknown line of action) and a couple.
Reactions of this group involve three unknowns, consisting of the two components of the force and the moment of the couple.

This reaction is caused by "fixed supports" which oppose any motion of the free body and thus constrain it completely, preventing both translation and rotation.

Examples of this type of support include
- A reinforced concrete floor/wall connection.
- A steel strap welded to a gusset plate.
- A timber pole embedded in a concrete base.
- A beam/column moment connection.

When the sense (direction) of an unknown force or couple is not known, no attempt should be made to determine the correct direction.
- The sense of the force or couple should be arbitrarily assumed.
- The sign of the answer that is obtained will indicate whether the assumption is correct or not.

Types of Loads
There are four fundamental types of loads that can act on a beam.

- Concentrated load (ex. beam-to-girder loads, column loads)

- Uniformly distributed load (ex. joists to beam, built-up roof, snow load, dead load, live load)

- Non-uniformly distributed load (ex. soil or water pressure behind a retaining wall)
• Pure moment (ex. starting torque on machinery)

7.2 Shear and Bending Moment
When a beam is subjected to applied loads, the beam must resist these loads to remain in equilibrium.
• To resist the applied loads, an internal force system is developed within the beam.
• Stresses and deflections in beams are the result of these internal forces (axial forces, shear forces, and moments).

It is convenient to “map” these internal actions (forces and moments).
• Diagrams are drawn to provide a complete picture of the magnitudes and directions of the forces and moments that act along the length of the beam.
• These diagrams are referred to as load (P), shear (V), and moment (M) diagrams.

Shear and moment diagrams can be drawn in three ways.
• By calculating values of shear and moment at various sections along the beam and plotting enough points to obtain a smooth curve. This procedure is time-consuming.
• By writing equations for the values of shear and moment.
• By drawing the diagrams directly, understanding the relationship between load, shear force and moment.

The two latter methods will be developed in the following sections of this chapter.

Consider the simply-supported beam with the concentrated load.
• The internal forces in member AB are not limited to producing axial tension or compression (as in the case of straight two-force members).
• The internal forces also produce “shear” and “bending.”
We can take a cut at any point along the length of the member to find the three internal actions.

If the entire beam is in equilibrium, then any portion of the beam is in equilibrium by the three internal actions.

\[ P \] - axial force

\[ V \] - shear force

\[ M \] - bending moment

A sign convention is necessary for shear and moment diagrams if the results obtained from their use are to be interpreted properly.

The bending moment in a horizontal beam is positive at sections for which the top fibers of the beam are in compression and the bottom fibers are in tension.
It is important to note that the sign convention used for shear and moment diagrams is different from the conventions used in writing the equations of equilibrium.

- When using the equations of equilibrium, forces directed up or to the right are positive; counterclockwise moments are positive and clockwise moments are negative.

- The new sign conventions are used only for showing the directions of the internal actions on free-body diagrams and for plotting the shear and moment diagrams.
Example Problems - Values of Shear and Moment at Specific Locations on a Beam

Given: The beam loaded as shown.

Find: V and M @ x = 0", 2", 4", 6", 8", 10", and 12"

Solution

First, find the reactions at the supports.

Σ \( M_B \) = 0 = -12 \( A_y \) + 12 (8) + 6 (4)
12 \( A_y \) = 96 + 24 = 120
\( A_y \) = +10.0 k \( A_y \) = 10.0 k \( \uparrow \)

Σ \( M_A \) = 0 = 12 \( B_y \) - 12 (4) - 6 (8)
12 \( B_y \) = 48 + 48 = 96
\( B_y \) = +8.0 k \( B_y \) = 8.0 k \( \uparrow \)

\( x = 0" \)
Σ \( F_y \) = 0 = 10 - V \( V = 10 \) k
Σ \( M_{cut} \) = 0 = -10 (0) + M \( M = 0 \) k-ft

\( x = 2" \)
Σ \( F_y \) = 0 = 10 - V \( V = 10 \) k
Σ \( M_{cut} \) = 0 = -10 (2) + M \( M = 20.0 \) k-ft

\( x = 4" \)
Σ \( F_y \) = 0 = 10 - V \( V = 10 \) k
Σ \( M_{cut} \) = 0 = -10 (4) + M \( M = 40.0 \) k-ft
\[ x = 4' \]
\[ \sum F_y = 0 = 10 - 12 - V \quad V = -2 \text{k} \]
\[ \sum M_{\text{cut}} = 0 = -10 (4) + 12 (0) + M \]
\[ M = 40.0 \text{k-ft} \]

\[ x = 6' \]
\[ \sum F_y = 0 = 10 - 12 - V \quad V = -2 \text{k} \]
\[ \sum M_{\text{cut}} = 0 = -10 (6) + 12 (2) + M \]
\[ M = 60 - 24 = 36.0 \text{k-ft} \]

\[ x = 8' \]
\[ \sum F_y = 0 = 10 - 12 - V \quad V = -2 \text{k} \]
\[ \sum M_{\text{cut}} = 0 = -10 (8) + 12 (4) + M \]
\[ M = 80 - 48 = 32.0 \text{k-ft} \]

\[ x = 8'' \]
\[ \sum F_y = 0 = 10 - 12 - 6 - V \quad V = -8 \text{k} \]
\[ \sum M_{\text{cut}} = 0 = -10 (8) + 12 (4) + 6 (0) + M \]
\[ M = 80 - 48 - 0 = 32.0 \text{k-ft} \]

\[ x = 10' \]
\[ \sum F_y = 0 = 10 - 12 - 6 - V \quad V = -8 \text{k} \]
\[ \sum M_{\text{cut}} = 0 = -10 (10) + 12 (6) + 6 (2) + M \]
\[ M = 100 - 72 - 12 = 16.0 \text{k-ft} \]

\[ x = 12'' \]
\[ \sum F_y = 0 = 10 - 12 - 6 - V \quad V = -8 \text{k} \]
\[ \sum M_{\text{cut}} = 0 = -10 (12) + 12 (8) + 6 (4) + M \]
\[ M = 120 - 96 - 24 = 0 \text{k-ft} \]
To simplify the calculations, use a free body diagram from the right side.

\[ x = 10' \]
\[ \sum F_y = 0 = V + 8 \quad V = -8 \text{ k} \]
\[ \sum M_{\text{cut}} = 0 = -M + 8 \quad M = 16.0 \text{ k-ft} \]

\[ x = 12' \]
\[ \sum F_y = 0 = V + 8 \quad V = -8 \text{ k} \]
\[ \sum M_{\text{cut}} = 0 = -M + 8 \quad M = 0 \text{ k-ft} \]

Note: The moment is zero at the supports of simply supported beams.
Example

Given: The beam loaded as shown.

Find: \( V \) and \( M \) @ \( x = 0'', 3'', 6'', 9'' \)

Solution

First, find the reactions at the supports.

\[ \sum M_B = 0 = -9A_y + \frac{1}{2} (6) (3 + (2/3) 6) + \frac{1}{2} (3) 6 (3 + (1/3) 6) \]

\[ 9A_y = 18 (7) + 9 (5) = 126 + 45 = 171 \]

\[ A_y = +19.0 \quad A_y = 19.0 \text{ k} \uparrow \]

\[ \sum M_A = 0 = 9B_y - \frac{1}{2} (6) (1/3) 6 - \frac{1}{2} (3) 6 (2/3) 6 \]

\[ 9B_y = 18 (2) + 9 (4) = 36 + 36 = 72 \]

\[ B_y = +8.0 \quad B_y = 8.0 \text{ k} \uparrow \]

\( x = 0'' \)

\[ \sum F_y = 0 = 19 - V \quad V = 19 \text{ k} \]

\[ \sum M_{\text{cut}} = 0 = -19 (0) + M \quad M = 0 \text{ k-ft} \]

\( x = 3'' \)

\[ \sum F_y = 0 = 19 - \frac{1}{2} (6) 3 - \frac{1}{2} (4.5) 3 - V \]

\[ V = 19 - 9 - 6.75 = 3.25 \text{ k} \]

\[ \sum M_{\text{cut}} = 0 = -19 (3) + \frac{1}{2} (6) (2/3) 3 + \frac{1}{2} (4.5) 3 (1/3) 3 + M \]

\[ M = 57 - 18 - 6.75 = 32.25 \text{ k-ft} \]

\( x = 6'' \)

\[ \sum F_y = 0 = 19 - \frac{1}{2} (6) 6 - \frac{1}{2} (3) 6 - V \]

\[ V = 19 - 18 - 9 = -8.0 \text{ k} \]

\[ \sum M_{\text{cut}} = 0 = -19 (6) + \frac{1}{2} (6) (2/3) 6 + \frac{1}{2} (3) 6 (1/3) 6 + M \]

\[ M = 114 - 72 - 18 = 24.0 \text{ k-ft} \]

7.11
\[ x = 6'' \]
\[ \sum F_y = 0 = V + 8 \quad V = -8.0 \text{ k} \]
\[ \sum M_{\text{cut}} = 0 = -M + 8 \quad M = 24.0 \text{ k-\text{ft}} \]

\[ x = 7.5' \]
\[ \sum F_y = 0 = V + 8 \quad V = -8.0 \text{ k} \]
\[ \sum M_{\text{cut}} = 0 = -M + 8 \quad M = 12.0 \text{ k-\text{ft}} \]

\[ x = 9'' \]
\[ \sum F_y = 0 = V + 8 \quad V = -8.0 \text{ k} \]
\[ \sum M_{\text{cut}} = 0 = -M + 8 \quad M = 0 \text{ k-\text{ft}} \]
7.3 Equilibrium Method for Shear and Moment Diagrams

One basic method used in constructing shear (V) and moment (M) diagrams is referred to as the equilibrium method.

- Specific values of V and M are determined from the equations of equilibrium.

Procedure

Determine how many sets of equations are needed to define shear and moment across the length of the beam.

- If the load is uniformly distributed or varies according to a known equation along the entire beam, a single equation can be written for shear (V) or moment (M).

- Generally, it is necessary to divide the beam into intervals bounded by abrupt changes in the loading (e.g. the end or beginning of a distributed load, a concentrated load or couple).

A free-body diagram is drawn representing the section of the beam of interest.

- An origin is usually selected at the left end of the beam for reference.

Equations for V and M are determined from free-body diagrams of portions of the beam.
Example Problems - Equilibrium Method for Shear and Moment Diagrams

Given: Beam loaded as shown.

Find: Write the V and M equations and draw the V and M diagrams.

Solution

One set of equations is required.

First, solve for the reactions at the supports.

\[ \sum M_B = 0 = -A_y L + w L (L/2) \]
\[ A_y L = w L^2 / 2 \]
\[ A_y = + w L/2 \quad A_y = w L/2 \quad \uparrow \]

\[ \sum M_A = 0 = + B_y L - w L (L/2) \]
\[ B_y L = w L^2 / 2 \]
\[ B_y = + w L/2 \quad B_y = w L/2 \quad \uparrow \]

Note: The beam is symmetrical and symmetrically loaded; thus, the reactions are symmetrical as well.

- Each reaction is equal to half of the total applied load (i.e. \( w L/2 \)).

\[ 0 < x < L \]
\[ \sum F_y = 0 = wL/2 - wx - V \]
\[ V = w (L/2 - x) \]

\[ \sum M_{cut} = 0 = -(wL/2) x + wx (x/2) + M \]
\[ M = -wx^2/2 + wLx/2 \]
\[ M = (wx/2) (L - x) \]
Example

Given: Beam loaded as shown.

Find: Write the V and M equations and draw the V and M diagrams.

Solution

Two sets of equations are required.

First, solve for the reactions at the supports.

\[ \sum M_B = 0 = -A_y L + P (L/2) \]
\[ A_y = P/2 \]

\[ \sum M_A = 0 = B_y L - P (L/2) \]
\[ B_y = P/2 \]

Note: The beam is symmetrical and symmetrically loaded; thus, the reactions are symmetrical as well.
• Each reaction is equal to half of the total applied load (i.e. P/2).

0 < x < L/2
\[ \sum F_y = 0 = P/2 - V \]
\[ V = P/2 \]
\[ \sum M_{\text{cut}} = 0 = - (P/2) x + M \]
\[ M = Px/2 \]

L/2 < x < L
\[ \sum F_y = 0 = P/2 - P - V \]
\[ V = - P/2 \]
\[ \sum M_{\text{cut}} = 0 = - (P/2) x + P (x - L/2) + M \]
\[ M = Px/2 - Px + PL/2 \]
\[ M = P/2 (L - x) \]
Problem 7.8 (p. 360)

Given: Beam loaded as shown.

Find: Write the V and M equations and draw the V and M diagrams.

Solution

Two sets of equations required.

First, solve for the reactions at the supports.

\[ \sum M_B = 0 = -6 A_y + \frac{1}{2} (240) 3 \left[ 3 + \left( \frac{1}{3} \right) 3 \right] \]
\[ + \frac{1}{2} (240) 3 \left( \frac{2}{3} \right) 3 \]
\[ 6 A_y = 360 (4) + 360 (2) \]
\[ 6 A_y = 1440 + 720 = 2160 \]
\[ A_y = +360 \text{ k} \quad A_y = 360 \text{ k} \uparrow \]

\[ \sum M_A = 0 = +6 B_y + \frac{1}{2} (240) 3 \left[ \left( \frac{2}{3} \right) 3 \right] \]
\[ + \frac{1}{2} (240) \left( \frac{3}{3} \right) \left[ 3 + \left( \frac{1}{3} \right) 3 \right] \]
\[ 6 B_y = 360 (2) + 360 (4) = 720 + 1440 = 2160 \]
\[ B_y = +360 \text{ k} \quad B_y = 360 \text{ k} \uparrow \]

Note: The beam is symmetrical and symmetrically loaded; thus, the reactions are symmetrical as well.

- Each reaction is equal to half of the total applied load.
  - Total load = \( \frac{1}{2} \times 240 \times 6 = 720 \text{ kips} \)
  - Each reaction = \( \frac{1}{2} \times 720 = 360 \text{ kips} \)

0' < x < 3'

Since the intensity of the distributed load varies, an equation is needed to define the intensity of the distributed load (p) as a function of position “x” along the beam.
Using the general equation of a line: \( p = m x + c \)

- Determine the slope \( m \).
  
  \[ m = \frac{(240 - 0)}{3} = 80 \]
  
  So, \( p = 80 x + c \)

- Next, determine the "y-intercept" value "\( c \)".

  Known points on the line include: when \( x = 0 \), \( p = 0 \) and when \( x = 3 \), \( p = 240 \).

  Substituting the second condition into the equation \( p = 80 x + c \):
  
  \[ 240 = 80 (3) + c \quad c = 240 - 240 = 0 \]

  Thus, \( p = 80 x + 0 \), and \( p = 80 x \)

Now write the equations for shear and moment.

  \[ \sum F_y = 0 = 360 - V - \frac{1}{2} p x \]
  
  \[ 0 = 360 - V - \frac{1}{2} (80x) x \]
  
  \[ V = 360 - 40x^2 \]

  \[ \sum M_{cut} = 0 = -360x + M + \frac{1}{2} p x (x/3) \]
  
  \[ 0 = -360x + M + \frac{1}{2} (80x) x (x/3) \]
  
  \[ M = +360x - 40/3 x^3 \]
  
  \[ M = 360x - 13.33 x^3 \]

**3’ < x < 6’**

Since the intensity of the distributed load varies, an equation is needed to define the intensity of the distributed load \( p \) as a function of position "\( x \)" along the beam.

Using the general equation of a line: \( p = m x + c \)

- Determine the slope \( m \).
  
  \[ m = \frac{(0 - 240)}{3} = -80 \]
  
  So, \( p = -80 x + c \)

- Next, determine the "y-intercept" value "\( c \)".

  Known points on the line include: when \( x = 3 \), \( p = 240 \) and when \( x = 6 \), \( p = 0 \).

  Substituting the first condition into the equation \( p = 80 x + c \):
  
  \[ 240 = -80 (3) + c \quad c = 240 + 240 = 480 \]

  Thus, \( p = -80 x + 480 \), and \( p = 480 - 80 x \)
Now write the equations for shear and moment.

\[ \sum F_y = 0 = 360 + V - \frac{1}{2} p (6 - x) \]
\[ 0 = 360 + V - \frac{1}{2} (480 - 80 x) (6 - x) \]
\[ V = -360 + (240 - 40 x) (6 - x) \]
\[ V = -360 + 1440 - 240 x - 240 x + 40 x^2 \]
\[ V = 40 x^2 - 480 x + 1080 \]

\[ \sum M_{cut} = 0 = 360 (6 - x) - M - \frac{1}{2} p (6 - x) (1/3) (6 - x) \]
\[ 0 = 360 (6 - x) - M - \frac{1}{2} (480 - 80 x) (6 - x) (1/3) (6 - x) \]
\[ M = 360 (6 - x) - (1/6) (480 - 80 x) (6 - x) (6 - x) \]
\[ M = 360 (6 - x) - (1/6) (480 - 80 x) (36 - 12 x + x^2) \]
\[ M = 2160 - 360 x - (1/6) (17,280 - 8640 x + 1440 x^2 - 80 x^3) \]
\[ M = 2160 - 360 x - 2880 + 1440 x - 240 x^2 + (80/6) x^3 \]
\[ M = 13.33 x^3 - 240 x^2 + 1080 x - 720 \]
7.4 Relationship Between Load, Transverse Shear, and Bending Moment

The methods presented previously for drawing shear and bending moment diagrams become increasingly cumbersome as the loading becomes more complex.

The construction of shear and bending moment diagrams can be simplified by understanding the relationships that exist between the load, shear force, and bending moment.

Relationships between Load and Shear Force

\[ \Sigma F_y = 0 = V - (V + \Delta V) - w \Delta x, \text{ where } w = w(x) \text{ and is constant for small } \Delta x. \]
\[ \Delta V = -w \Delta x \]
\[ \Delta V/\Delta x = -w \]

In calculus, as \( \Delta x \rightarrow 0 \), the above expression takes the following form.
\[ \frac{dV}{dx} = -w \]

The equation shows that the slope of the shear diagram is equal to the intensity of the loading at that point, times minus one.

If we integrate this expression between two points
\[ dV = -w \, dx \]
\[ \int dV = - \int w \, dx \]
\[ V_1 - V_2 = - \int w(x) \, dx \]

The equation shows that the change in shear between two points is equal to the area under the load diagram, times minus one.

These equations are not valid under a concentrated load.
- The shear diagram makes a jump at the point of a concentrated load.
Relationships between Shear Force and Bending Moment

From the free body diagram,

\[ \sum M_R = 0 = -M - V \Delta x + w (\Delta x)^2/2 + M + \Delta M \]

\[ \Delta M = V \Delta x - \frac{1}{2} w (\Delta x)^2 \]

\[ \Delta M / \Delta x = V \]

In calculus, as \( \Delta x \to 0 \), the above expression takes the following form.

\[ dM/dx = V \]

The equation shows that the value of the slope on the bending moment diagram is equal to the height of the shear diagram at that point.

- When the shear force is zero, the slope on the bending moment diagram is zero.
- Zero slope on the moment diagram corresponds to a point of a maximum bending moment (either the absolute maximum or a local maximum).

If we integrate this expression between two points

\[ \int dM = \int V \, dx \]

\[ M_1 - M_2 = \int V(x) \, dx \]

The equation shows that the change in bending moment between two points is equal to the area under the shear diagram.

These equations are not valid under a concentrated couple.

- The bending moment diagram makes a jump at the point of a concentrated couple.
7.5 **Semigraphical Method for Load, Shear and Moment Diagrams**

The method presented in the previous section is referred to as the *semigraphical method*.

- These expressions can be used to draw the shear and moment diagrams.
- These expressions can be used to compute values of shear and moment at various points along the beam.

Before proceeding, let’s review basic curves and curve relationships.

**Zero degree curve**
- The line is horizontal (or has a constant height).
- May represent the area under a uniformly distributed load.
- May represent the rectangular area of the shear diagram.

\[
\text{Area} = a \times x
\]

**1st degree curve**
- The line is straight with a constant slope.
- May represent the area under a triangular loading.
- May represent the triangular area of the shear diagram.

\[
\text{Area} = \frac{1}{2} b \times x
\]

**2nd degree curve**
- The line takes the form of a parabola.
- May represent the area of a shear diagram.

\[
\text{Area} = \frac{c x}{3}
\]
General considerations for drawing V and M diagrams.

- For a simply supported beam, the shear at the ends must equal the end reactions.
- For a simply supported beam, the moment at the ends must be zero.
- Shear and moment are both zero at the free end of a beam.
- For a fixed support, the reactions at the support are equal to the shear and moment values at the support.
- A concentrated load produces a “jump” in the shear diagram of the same magnitude and direction as the concentrated load.
- A concentrated couple produces a “jump” in the moment diagram of the same magnitude as the concentrated couple. The direction of the “jump” should be determined by analysis.
- At points where shear is zero, moment is a maximum.
Example Problems - Shear and Moment Diagrams by the Semigraphical Method

**Given:** The beam loaded as shown.

**Find:** Draw the shear and moment diagrams.

For the shear diagram

\[0 < x < L\]

\[
\text{Area under the load diagram } = + w L
\]

\[
\Delta V = - \text{Area } = - w L
\]

At the right end of the beam

\[V_2 = V_1 + \Delta V\]

\[= w L/2 - w L = - wL/2\]

For the moment diagram

\[0 < x < L/2\]

\[
\text{Area under the shear diagram } = + \frac{1}{2} \left(\frac{1}{2} w L\right) (L/2) = w L^2/8
\]

\[
\Delta M = \text{Area } = + w L^2/8
\]

At mid-span

\[M_2 = M_1 + \Delta M\]

\[= 0 + w L^2/8 = w L^2/8\]

**Note:** At \(x = L/2\), \(V = 0\), and \(dM/dx = 0\) (there is a horizontal tangent on the moment diagram).
Example

**Given:** The beam loaded as shown.

**Find:** Draw the shear and moment diagrams.
Example

Given: The beam loaded as shown.

Find: Draw the shear and moment diagrams.
Example

Given: The beam loaded as shown.

Find: Draw the shear and moment diagrams.

Find the reactions at the supports.

\[ \sum M_D = 0 = -18 A_y + \frac{1}{2} (4) 12 [6 + (1/3) 12] + 20 (3) - 120 \]
\[ 18 A_y = 240 + 60 - 120 = 180 \]
\[ A_y = + 10.0 \text{ k} \quad A_y = 10 \text{ k} \uparrow \]

\[ \sum M_A = 0 = -\frac{1}{2} (4) 12 (8) - 20 (15) - 120 + 18 D_y \]
\[ 18 D_y = 192 + 300 + 120 = 612 \]
\[ D_y = + 34.0 \text{ k} \quad D_y = 34.0 \text{ k} \uparrow \]

In order to draw the first part of the moment diagram, the location for the point of maximum moment (i.e. \( V = 0 \) kips) needs to be determined so that the area under the shear diagram may be calculated.

• Write an equation for shear between \( x = 0' \) and \( x = 12' \).

\[ \sum F_y = 0 = 10 - \frac{1}{2} (x) (x/3) - V \]
\[ V = 10 - x^2/6 \]

Let \( V = 0 \), thus \( 0 = 10 - x^2/6 \)
\[ x^2 = 60.0 \]
\[ x = 7.75' \]

• The change in moment from \( x = 0' \) to \( x = 7.75' \) is equal to the area under the shear diagram.

\[ \Delta M = (2/3) 10 (7.75') = 51.7 \text{ kip-ft} \]
• Determine the value of moment at \( x = 12' \) and plot this value on the moment diagram.

  - The area under the portion shear diagram between \( x = 7.75' \) and \( x = 12' \) cannot be calculated using standard formulas since there is no horizontal tangent at \( x = 7.75' \) or at \( x = 12' \).

  - Take a cut at \( x = 12' \), draw the free body diagram, and write the equilibrium equation to determine the value of moment.

  - Then plot this value on the moment diagram.

\[
\sum M_{\text{cut}} = 0 = M_B - 10 \times 12 + \frac{1}{2} \times 12 \times 4 \times (1/3) \times 12
\]

\[ M_B = 120 - 96 = 24.0 \text{ kip-ft} \]

The concentrated couple at point C causes the moment diagram to jump up by 120 kip-ft (from -18 kip-ft to +102 kip-ft).

• Since the applied moment is acting clockwise, the internal resisting moment increases, thus the jump "up" on the moment diagram.
Problem 7.8 (p. 360)

Note: This problem was previously worked by writing the equations for shear and moment.

Given: Beam loaded as shown.

Find: Draw the shear and moment diagrams.
Problem 7.13 (p. 362)

Given: The beam loaded as shown.

Find: Draw the shear and moment diagrams.

Solution

Find the reactions at the supports.

FBD: Member CD

\[ \sum M_C = 0 = 8D_y - 4(8)4 \]
\[ D_y = +16 \text{ k} \quad D_y = 16 \text{ k} \uparrow \]

\[ \sum M_D = 0 = -8C_y + 4(8)4 \]
\[ C_y = +16 \text{ k} \quad C_y = 16 \text{ k} \uparrow \]

FBD: Member ABC

\[ \sum M_A = 0 = 16B_y - 4(20)10 - 16(20) \]
\[ B_y = +70 \text{ k} \quad B_y = 70 \text{ k} \uparrow \]

\[ \sum M_B = 0 = -16A_y + 4(20)6 - 16(4) \]
\[ A_y = +26 \text{ k} \quad A_y = 26 \text{ k} \uparrow \]

Note: Horizontal reactions are statically indeterminate, but are not needed to draw the shear and moment diagrams.

Calculation of areas under the load diagram

A to B: Area = +16 (4) = +64
B to C: Area = +4 (4) = +16
C to D: Area = +4 (8) = +32

Calculation of areas under the shear diagram.

\[ A_1 = \frac{1}{2} (26) 6.5 = 84.5 \]
\[ A_2 = \frac{1}{2} (-38) 9.5 = -180.5 \]
\[ A_3 = \frac{1}{2} (32 + 16) 4 = 96.0 \]
\[ A_4 = \frac{1}{2} (16) 4 = 32.0 \]
\[ A_5 = \frac{1}{2} (-16) 4 = -32.0 \]